CHAPTER SIX

Reasoning and Belief in Victorian Mathematics

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Given the great vitality and immense progress of mathematics in the nineteenth century, even a purely technical history of the discipline stripped of its larger intellectual and social context would not soon run out of material. Existing mathematical fields diversified and new fields arose in response to significant breakthroughs. The eastern European mathematicians János Bolyai and Nikolai Ivanovich Lobachevsky formulated non-Euclidean geometry in the Victorian period, a set of principles counterintuitive to normal human experience that led to a complete redefinition of this most ancient of mathematical pursuits. Mathematics and its associated methodologies also expanded into realms of knowledge other than the natural sciences (where they had been especially at home in physics and astronomy), often through pioneering work by theorists who began their careers as mathematicians but who branched out later in life. At the same time that this move outward occurred, there was a move inward in nineteenth-century mathematics. Concerns about the foundations of the discipline—an interest in the fundamental nature of mathematical knowledge and the process whereby mathematicians come to conclusions—occupied a significant portion of the research agenda. In Great Britain the adoption of Gottfried Leibniz's more suggestive form of the calculus over Isaac Newton's method, although far from rapid and unchallenged, eventually led to an acceleration of mathematical discovery in the British isles, and it signalled a greater integration with Continental mathematics that would ultimately result in the first major international conferences toward the end of the century.

British interest in the formal aspects of mathematics was particularly apparent in the growing interest in mathematical, or symbolic, logic. As George Boole (1815–64) summarised the nature of this critical field of pure mathematics, it was 'not of the mathematics of number and quantity alone, but of mathematics in its larger ... truer sense, as universal reasoning expressed in symbolical forms, and conducted by laws, which have their ultimate abode in the human mind'.[1] Alfred North Whitehead (1861–1947), following decades of British work on Boole's project, would provide an even stronger statement about mathematics at the end of the Victorian period:

> Mathematics in its widest signification is the development of all types of formal, necessary, deductive reasoning. The reasoning is formal in the sense that the meaning of propositions forms no part of the investigation. The sole concern of mathematics is the inference of proposition from proposition ... The ideal of mathematics should be to erect a calculus to facilitate reasoning in connection with every providence of thought, or external experience, in which the succession of thoughts, or of events can be definitely ascertained and precisely stated. So that all serious thought which is not philosophy, or inductive reasoning, or imaginative literature, shall be mathematics developed by means of a calculus.[2]

Whitehead thus envisioned mathematics in 1898 as a discipline that applied to a much larger region of human thought than it did before the nineteenth century. No longer was mathematics simply about familiar elements such as the numbers with which we count or the arc of comets through the solar system. Mathematics could be responsible for sound reasoning and conclusions regardless of the topic, and thus encroached on, or even assumed territory once commanded by philosophy.

While research fields abounded and British mathematicians of this era studied and contributed to many of them, they showed a special interest in this abstract field of symbolic logic, and they pursued it with special intensity. Many of the pioneers in this field were British, a remarkable three generations that furthered the association between British thought and logic while creating a new mathematical field in concert with European counterparts: Boole and Augustus De Morgan (1806–71), William Stanley Jevons (1832–85) and John Venn (1834–1923), Bertrand Russell (1872–1970) and Whitehead. The 1840s and 1850s saw the ground-breaking publication of Boole's *The Mathematical Analysis of Logic, being an Essay towards a Calculus of Deductive Reasoning* (1847) and *An Investigation of the Laws of Thought on which are Founded the Mathematical Theories of Logic and Probabilities* (1854), as well as De Morgan's *Formal Logic* (1847). Jevons, a student of De Morgan's at University College London, began his career by formulating his own symbolic logic (*Pure Logic*, 1864), which led to his landmark treatise *The Principles of Science* (1874), and he continued to work

[1] G. Boole, *Studies in Logic and Probability* (La Salle, 1952), p. 195.

[2] A. N. Whitehead, *A Treatise on Universal Algebra* (New York, 1960), pp. vi, viii.

in the field as he carried its methods into economics and the social sciences in general. Venn expanded upon Boole's theories in two critical texts in the 1880s, *Symbolic Logic* (1881) and *The Principles of Empirical Logic* (1889), in the process inventing the diagrams of overlapping shapes that would come to bear his name. *Principia Mathematica* (1910–13), in which Russell and Whitehead equated logic and mathematics at the deepest level possible, was a culmination of the innovative mathematical research of the Victorian age. Before this seminal collaboration, Russell and Whitehead had independently penned monographs exploring mathematical logic (Russell's *The Principles of Mathematics*, 1903; Whitehead's *A Treatise on Universal Algebra*, 1898). Though far from the totality of British mathematics in the nineteenth century, these were among the most highly influential figures in Victorian mathematical circles due to their wide-ranging thought (e.g., Boole penned an important treatise on differential equations in addition to his logical work) and institutional positions (especially true for Augustus De Morgan and John Venn).

While intellectual historians have given a great deal of attention to Russell and Whitehead, the embryonic theories and methods of their predecessors provide useful insights into the state of mathematical knowledge in the Victorian age and its relationship to other disciplines and other elements of Victorian culture in general. Exploring such relationships demands a wider view than that of a purely technical history, of course; questions about mathematical research agendas naturally lead to broader questions about the goals and motivations of Boole, De Morgan, Jevons, and Venn. Why did mathematical logic flourish in the British isles in the second half of the nineteenth century, and why did British mathematicians pursue this particular region of their discipline with such passion? What motivated the founders of mathematical logic, Boole and De Morgan, and why were promising young British mathematicians eager to embrace and extend their work? More specifically, why did these mathematicians criticise words and common methods of reasoning as notoriously unreliable and seek to replace them with what they believed were far more exact symbolic replacements and logical processes? In a larger sense, what did these processes say about mathematical reasoning and knowledge? How widely applicable was mathematics? Was mathematical knowledge unlimited or limited? Objective or subjective? Perfect or merely approximate?

While the Victorians Boole, De Morgan, Jevons, and Venn had their differences, they shared a great curiosity about such questions and their broader significance. Tellingly, they also shared important experiences and concerns that had a clear impact on the way they thought about these questions. By tackling logic these thinkers set mathematics on a collision course with schools of philosophy both ancient and modern, and they had to struggle against the calcification of logic in educational curricula. This was not unusual in the history of Victorian mathematics; Victorian geometers faced an uphill battle against the long-standing dominance of Euclid in mathematical education at Cambridge.[3] In addition to these institutional difficulties, the mathematical logicians addressed theological and sectarian debates both indirectly and directly. The reader of works of Victorian symbolic logic is struck by the high frequency and extra-mathematical overtones of words such as 'clarity', 'conviction', 'certainty', 'fallibility', and 'belief', and cannot help but notice how certain mathematical investigations functioned as proxies for religious inquiries. It is also notable that without exception the mathematical logicians drifted away from the Church of England and toward what they saw as more ecumenical expressions of spirituality, and had social agendas marked by liberalism, cosmopolitan internationalism, and pacifism. Unsurprisingly, much of their pioneering mathematical work took place outside of the traditional English stronghold of mathematics (and until 1871 an institution that mandated orthodoxy), Cambridge University. De Morgan taught at University College London; Boole at Queen's College, Cork; Jevons split his time between the University of Manchester and University College London. Only John Venn, from a prominent line of Cantabrigians, taught at Cambridge and took holy orders, though he later resigned those orders.

Given the contentiousness and upheaval of early Victorian religious, social, and political life, Augustus De Morgan's quest for a 'logic of relations' took on much larger connotations, and a mathematical logic presented to George Boole a possible way out of the 'idle disputation' and 'wordy wrangling' that he saw as the unfortunate hallmark of his day.[4] Theological factions, as well as political and social ones (often intertwined with the religious sects), and their frequently heated, turbulent relationships made a cool, calm system of logic based on mathematics exceedingly attractive to these figures. Although Boole and De Morgan had disparate views of the fundamental nature of mathematical knowledge, both saw their creation of a symbolic logic as a social act as well as a technical achievement. A precise understanding of terms and the relationships between them would allow for a more nuanced and thus less contentious dialogue between opposing groups,

[3] See J. Richards, *Mathematical Visions: the Pursuit of Geometry in Victorian England* (San Diego, 1989).

[4] G. Boole, 'Lines written in the autumn of 1846', Dublin, Royal Irish Academy, MS 12.K.45, f. vii (hereafter referred to as RIA).

they believed. Support for every conclusion would have to come through logical methods accepted even by those who were disinclined to the position in question. Boole and De Morgan saw mathematics as particularly well suited to this task. After all, it was an international language that transcended so many human-drawn lines. De Morgan's insignia for the London Mathematical Society, founded in 1865, neatly encapsulated this idea. Based on a triangular symbol from Euclid's *Elements*, it displayed the Christian, Jewish, and Muslim years on its three sides—a symbol of the unity of peoples across religious faiths.

De Morgan expressed in words as well as symbols the ecumenism that often characterised researchers in mathematical logic, and that was particularly important in the genesis of this field in Great Britain. Born in India, and after adolescence disdainful toward his parents' evangelicalism, De Morgan called himself a 'Briton unattached' and a 'Christian unattached'.[5] He was unable to take his master's degree at Trinity College, Cambridge because of an unwillingness to subscribe to the thirty-nine articles of the Church of England,[6] and later referred to subscriptions as 'deadly poison' which 'foster[ed] every kind of dishonesty' because in the realm of private conscience many of the faithful did not agree with the doctrines of their church.[7] De Morgan's favourite quip about the Bible summarised his contempt for doctrinaire sectarians: 'One day at least in every week/The sects of every kind/Their doctrines here are sure to seek/And just as sure to find.'[8] Less flippant and more heartfelt was his declaration of religious heterodoxy to his mother. In early adulthood he wrote her a letter distancing himself from Church of England orthodoxy 'because I see in all that is orthodox a lack of that charity which Paul considers as more essential than everything else, coupled with what virtually amounts to a claim of infallibility'.[9] Instead of pursuing a career at Cambridge or Oxford, he instead signed on as the first professor of mathematics at the new University College London in 1828, cherishing its novel non-denominational status. He hoped for the realisation of Lord Brougham's prediction that the new university would 'do more to crush bigotry and intolerance than all the Bills [we] will ever see carried, at

[5] De Morgan to W. R. Hamilton, 2 Feb. 1852, Dublin, Trinity College Library, Hamilton papers, 1493/541; Sophia Elizabeth De Morgan, *Memoir of Augustus De Morgan* (London, 1882), p. 86.
[6] A. De Morgan, autobiographical sketch written in the third person, London, British Library, MS 28509, f. 421.
[7] De Morgan to Hamilton, 27 July 1852, in R. Graves, *Life of Sir William Rowan Hamilton* (3 vols., Dublin, 1885), iii, p. 395.
[8] De Morgan to Hamilton, 1 Sept. 1852, in Graves, *Hamilton*, iii, p. 410.
[9] S. De Morgan, *Memoir*, p. 142.

least until a Reform happens'.[10] De Morgan was an early supporter of the Catholic Emancipation Bill and an eager mentor of dissenting Christian as well as Jewish students.

De Morgan's friend George Boole found himself in a more unsettling non-denominational educational experiment during the time he was developing mathematical logic. Indeed, as a shy, distressed young faculty member in Cork, Ireland, the 'father of pure mathematics' (as Russell called Boole[11]) is perhaps a more interesting case study for understanding the relationship between work on the foundations of mathematical reasoning in the Victorian age and the ideologies and experiences of its creators. Throughout his life Boole was fond of talking about the unity of all human beings on a higher plane, above the lines drawn by culture, politics, and religion. As one of his poems declared in 1848, a year after his publication of *The Mathematical Analysis of Logic*, 'Oh, too long sever'd in the thought and speech/Of mortal men, too oft as rivals set/Who kindred are, and in firm union met/One consummation in the Heavens shall reach.'[12] Desperately concerned with forging interdenominational agreement, he was drawn to the idea of a highly abstract form of mathematics that might have some very real implications for the world outside his study.

While others argued over the nature of the Holy Trinity or the eucharist, or engaged in charged debates about the meaning of single lines or words from the Bible, George Boole developed an increasing distaste for the state of religious affairs in Great Britain. Although he was raised in a family that subscribed to Low Church Anglican principles, he fell away from the Church of England in his early adulthood, eventually settling on a faith that came closest to Unitarianism, closely informed by philosophical idealism. (There are also hints that he had an affinity toward Judaism, since to him it represented a pure, 'pre-sectarian' religiosity.[13]) Boole's mature faith was difficult to define because he found such religious definitions to be based, more often than not, on theological conjectures and nebulous terms. Instead, the best indicator of this faith came from his actions rather than his adherence to specific tenets or formulations. He enjoyed reading all kinds of religious works, including the lives of saints (Roman Catholic as well as Protestant), eastern

[10] Quoted. in A. Desmond, *The Politics of Evolution: Morphology, Medicine, and Reform in Radical London* (Chicago, 1989), p. 25.

[11] B. Russell, n.d., RIA, untitled tribute to G. Boole.

[12] G. Boole, 'Sonnet XIII', 3 July 1849, RIA, MS 12.K.45, f. vi.

[13] Boole to J. Hill, 30 May 1837, Cork, University College Library, Boole papers, 1/221(6) (hereafter referred to as UCC). Boole to De Morgan, 4 Nov. 1861, London, University College Library, De Morgan papers, MS Add. 97.

and western mystic philosophy, the Hebrew and Christian Bibles, writings by puritan divines and church fathers, and Christian and Jewish Prayer Books. He could be found on any given Sunday in services at a Society of Friends house, a Roman Catholic or Church of England cathedral, or a Unitarian communion—as long as the liturgy included music. 'Once a man thinks himself bound to a settled creed, it seems as if truth, faith, and charity become impossible to him, except in so far as he evades his creed', Boole told his wife.[14] In short, he was a spiritual omnivore who resisted and deplored the notion of religious sects.

Unfortunately, as a teacher in England and then a professor in Ireland Boole found himself perpetually in the middle of sectarian firestorms. In his first teaching job in the 1830s, at a Methodist school in his home town of Lincoln, the young mathematician became the centre of attention when the Methodist community discovered that he was not a Methodist himself. As the Methodists began to pray publicly for his conversion, a highly uncomfortable Boole decided he would be better off somewhere else.[15] He sought an environment that would downplay partisanship and promote unity across sectarian boundaries, and he believed that he had found it in the late 1840s when he was appointed the first professor of mathematics at the recently created Queen's College in Cork, Ireland (now known as University College, Cork). With characteristic idealism he thought that a new institution of higher education in a city roughly split between Roman Catholics and Protestants would show his nation and the world that sectarian coexistence—and even friendship—was a real and desirable possibility. Boole told a Lincoln audience before setting off for his new position that a primary mission of QCC was 'the bringing together and associating of the Catholic and the Protestant youth', and that it 'would contribute to the harmony of Ireland, and that [the Queen's colleges] would in their internal management prove models of that peace and harmony which they recommended'.[16]

The experiment did not turn out, however, as Boole had hoped. Catholic/Protestant and Irish/British tensions actually intensified during Boole's tenure in Cork from the late 1840s until his death in 1864, only reinforcing his sense that the greatest trouble of his age was a tremendous factionalism that threatened to swamp civil society and destroy Christian brotherhood. In response to the founding of the Queen's colleges in Ireland,

[14] M. E. Boole, 'Home-side of a scientific mind', in *Collected Works,* 4 vols. (London, 1931), i, p. 3.
[15] J. Dyson to M. Boole, undated letter, UCC, 1/256.
[16] Lincoln, Lincolnshire Central Library, Local history collection, MS UP 9663, undated newspaper clipping, probably from *The Stamford Record* (hereafter referred to as LCL).

Pope Pius IX began to consider funding a countervailing system of Roman Catholic colleges. Following the summer 1850 session at the Synod of Thurles, where the primate of Ireland, Dr Cullen, warned of the 'godlessness' of the Queen's colleges,[17] local priests began advising their flocks to turn away from the schools. As he returned to campus for his second year at QCC, Boole discovered that his Catholic students were extremely apprehensive and increasingly absent. He wrote to Augustus De Morgan in October 1850 describing in horror 'the storm of religious bigotry which is at this moment raging around us here'.[18] Adding fuel to the fire was the publication in 1850 of a work by one of Boole's Protestant housemates that seemed to undermine the authority of the pope by exploring the misty circumstances of pontifical succession in the early Christian era.[19] With a president, Sir Robert Kane, who was impolitic and unable to maintain a sense of fairness and balance at QCC, and with the famine heightening the sense of injustice among the Roman Catholic populace, the taciturn Boole frequently and painfully found himself at the wrong end of coarse diatribes at formal dinner parties.[20] The mathematician's early concern that the new college would fail in its noble mission 'if that charity which was the essence of true religion was lost amid the turmoil of theological disputation' quickly became a reality.[21]

Sadly for Boole, it did not seem possible to escape from this sectarian strife by returning to England. The autumn of 1850 saw the rise of strong anti-Catholic and anti-papist sentiments across England in response to Pius's attempt to re-establish the Roman Catholic hierarchy there. Lord Russell questioned the intent of the pope and attacked the tractarians for complicity. A wave of popular anti-Catholicism crested on Guy Fawkes Day, 5 November, with 'No Popery' riots and physical assaults on Roman Catholic priests. In that same month George Boole received an offer to teach in Manchester, but he politely declined, worrying in a letter to his sister about 'the ill feeling which is springing up between Protestant and Catholic in England'.[22] Later in the 1850s Boole received an offer to teach mathematics at Oxford University, yet despite the tense situation in Cork he feared worse back in England because of the Oxford Movement and its vocal antagonists.

[17] Quoted. in W. Ralls, 'The papal aggression of 1850: a study in Victorian anti-Catholicism', in G. Parsons (ed.), *Religion in Victorian Britain* (Manchester, 1988), p. 127.

[18] Boole to De Morgan, 17 Oct. 1850, in G. C. Smith (ed.), *The Boole–De Morgan Correspondence* (Oxford, 1982), p. 38.

[19] L. R. de Vericour, *An Historical Analysis of Christian Civilisation* (London, 1850), p. 18.

[20] G. Boole to M. E. Boole, 20–5 Mar. 1850, UCC, 1/150.

[21] LCL, undated newspaper clipping (probably from *The Stamford Record*).

[22] G. Boole to M. Boole, 18 Nov. 1850, UCC, 1/46.

'He would be expected to take one side or the other in [the] theological controversy' between High and Low Churchmen, his wife later explained, 'the life of a man who would be a partisan of neither side might be made very uncomfortable'.[23] Boole nervously delayed his response and ultimately put in a half-hearted application that was rejected.

In retrospect Boole might have been better off taking either position in England as factionalism increasingly disrupted QCC. When Boole tried to admonish Kane in the mid-1850s to instill a more harmonious, non-denominational spirit, he was rebuffed with a disturbing counter-accusation of being sectarian himself. 'When the most reasonable and temperate efforts to bring about a better state of things expose a man to the charge of faction and subject them to the frown of power, I do not see what but ruin can be expected', Boole lamented to a friend.[24] Ultimately, the Roman Catholic unease with the Queen's colleges led to a final papal condemnation of them in the spring of 1857 and the establishment of a rival university system. Catholic students at QCC were confronted with the option of leaving the school or facing the possibility of excommunication, a terrible and tragic dilemma in Boole's mind.[25] Near the end of his life, while still at work in Cork, Boole complained bitterly that 'the Roman Catholic Priesthood seem to have been doing all they can to preach disloyalty. Between them and a bigoted Calvinistic Protestant population this is a country which does not on the whole present the most favourable picture of Christianity.'[26] The Cork experiment had failed; far from diminishing sectarian feelings, the new college had actually magnified them.

Unsuccessful in his mostly timid public entreaties, George Boole more productively channelled his drive to forge common ground between factions by developing a logic based on mathematical principles. Mathematics, Boole believed, was a divine language that transcended human differences and vagaries, and thus was perfectly suited to be the basis of a new, dispassionate system of arbitration. A mathematical logic might ease tensions by undermining the excessive verbiage and fatuous arguments that had flourished in the sectarian atmosphere of the nineteenth century. In doing so, Boole modestly proposed, this symbolic method could 'render service in the investigation of social problems'.[27] In *The Laws of Thought* (1854) Boole emphasised

[23] M. E. Boole, 'Home-side,' p. 2.
[24] G. Boole to W. Brooke, 18 June 1855, UCC, 1/161(a).
[25] G. Boole to M. Boole, 25 May 1857, UCC, 1/125.
[26] G. Boole to A. T. Taylor, 26 Jan. 1860, UCC, 1/232.
[27] G. Boole, *An Investigation of the Laws of Thought on which are Founded the Mathematical Theories of Logic and Probabilities* (London, 1854), pp. 20–1.

the inherent neutrality of his mathematical system, which could be established without regard for intent or content—it involved objects and processes that were completely independent of all viewpoints, tenets, debaters, or interest groups. 'In employing the method of this treatise, the order in which premises are arranged, the mode of connexion which they exhibit, with every similar circumstance, may be esteemed a matter of indifference, and the process of inference is conducted with a precision which might almost be termed mechanical,' he proudly explained.[28] By replacing tenuous arguments and nebulous terms with more exact mathematical representations, and by analysing the relationships between such representations, mathematical logic could directly challenge the 'wordy wrangling' and 'idle disputation' that Boole loathed.

Published and unpublished sources show that Boole thought that his mathematical logic would be particularly useful in deflating overreaching theological declarations and exposing the often tautological or vague meanings of tightly held or exclusionary religious dogmas. 'I do think that when we know all the scientific laws of the mind we shall be in a better position for a judgment on its metaphysical questions', Boole wrote to Augustus De Morgan as he worked on his system.[29] To prove his point, in chapters of *The Laws of Thought* Boole analysed the much debated theological doctrines of Spinoza and Samuel Clarke, ultimately finding them empty and unworthy of serious discussion or controversy. Boole's notebooks contain attempts to deconstruct passages from John Henry Newman, Plato, the Hebrew and Christian Bibles, and contemporary writings on theodicy using his symbolic method.[30] For those who felt this dissection of theological arguments was unwarranted or unnecessary, Boole responded that 'the necessity of a rigorous determination of the real premises of a demonstration ought not to be regarded as an evil; especially as, when that task is accomplished, every source of doubt or ambiguity is removed'.[31] The carefully marshaled Xs and Ys of a mathematical system could replace the unclear and unhelpful terms, and the often suspect logic, that only served to draw lines between human beings, particularly in religion. 'Theology always has been, and always will and must be, reformed from the outside, and very much from the side of science', Mary Everest Boole wrote about her husband's motivation.[32]

[28] Boole, *Laws of Thought*, pp. 185–6.
[29] Boole to De Morgan, 8 Oct. 1852, in Smith, *Boole–De Morgan Correspondence*, p. 62.
[30] G. Boole, London, Royal Society of London, Boole papers, C1.6, C.35–7.
[31] Boole, *Laws of Thought*, pp. 185–6.
[32] M. E. Boole, 'Home-side', p. 7.

George Boole thought that a mathematical logic might provide salvation from the sectarian strife of the Victorian age.

Despite the technical benefits of the symbolic logic of George Boole and Augustus De Morgan, as well as the passionate ecumenical motivation behind it, their new method was hidden for a long time in the shadow of a prominent non-mathematician. While Boole and De Morgan published their initial research on the mathematical principles of logic in 1847, this was just four years after the publication of John Stuart Mill's landmark *System of Logic* (1843), which swiftly became one of the most important texts on the subject and a standard treatise in classes on logic, moral science, and philosophy. Mill, like Boole and De Morgan, had realised that it was high time for the ancient science of logic to undergo a major revision—Aristotle was still a major touchstone at the beginning of the Victorian period—yet from there, their methodologies diverged enormously. Extending and evolving an optimistic strain of British scientific philosophy from the early modern period and the thought of Francis Bacon, Isaac Newton, and John Locke, Mill was confident in the ability of the human mind to reach certain conclusions through the use of experience and the process of induction. This process would not replace the ancient logic based on the syllogism, but supplement it by providing a logical framework for sciences both natural and human.

Mill's reputation and his imposing *Logic* clouded the arrival of mathematical logic and its associated advances. 'I have often deplored the fact that though these works [of Boole] were published in the years 1847 and 1854, the current handbooks, and even the most extensive treatises on logic, have remained wholly unaffected thereby', Stanley Jevons lamented to the Royal Society in 1870.[33] That John Stuart Mill should eclipse George Boole and mathematical logic infuriated Jevons, the greatest proponent of Boole's system. Finding Mill's mind to be 'essentially illogical', Jevons wrote a series of articles and books in the 1870s and 1880s that tried to debunk Mill's logic and promote the ideas of the mathematicians instead.[34] He was outraged that he had to teach Mill's *Logic* in his courses on moral science while the works of his mentor Augustus De Morgan and his intellectual forerunner Boole languished. 'I will no longer consent to live silently under the incubus of bad logic and bad philosophy which Mill's Works have laid upon us', Jevons bemoaned in 1864, 'He has expressed unhesitating opinions, and his sayings are quoted by his admirers as if they were the oracles of a perfectly wise and

[33] W. S. Jevons, *Pure Logic and Other Minor Works* (London, 1890), p. 171.
[34] Ibid., p. 201.

logical mind.'[35] Like others interested in the new mathematical methods, Jevons was captive to the standard curriculum, which enshrined certain works and ruthlessly excluded others. With clear anger Jevons did not mince words regarding the role of Mill in the curriculum: 'For the last fourteen years I have been compelled, by the traditional requirements of the University of London, to make [the] works [of Mill] at least partially my text-books in lecturing ... Nothing surely can do so much intellectual harm as a body of thoroughly illogical writings, which are forced upon students and teachers by the weight of Mill's reputation, and the hold which his school has obtained upon the universities.'[36] Jevons ultimately quit teaching in 1880 so that he did not have to lecture any more on topics that seemed dated and utterly inferior to the newer subjects he admired.[37]

What Mill had failed to account for in the eyes of most of the Victorian mathematical logicians was the confusion of the very words we use to describe experience in the process of induction, as well as the often faulty methods we use to make connections and conclusions. Augustus De Morgan spent a great deal of his life deconstructing flawed reasoning and the inadequate language that almost always went along with it. 'The growth of inaccurate expression' that has arisen from a poor understanding of language and logic, De Morgan proclaimed in his groundbreaking work *Formal Logic* (1847), 'gives us swarms of legislators, preachers, and teachers of all kinds, who can only deal with their own meaning as bad spellers deal with a hard word, put together letters which give a certain resemblance, more or less as the case may be'.[38] As De Morgan cautioned the audience at the first meeting of the London Mathematical Society in 1865, 'If we do not attend to extension of language, we are shut in and confined by it.'[39] Mathematical logic, on the other hand, had clearer terms and the benefit of rigorous mathematical processes. Much of *Formal Logic* recounts the problems of human reasoning, which De Morgan felt had reached a low point in his age, with the vague application of language greatly intensifying partisanship. A reasonable, moderate disposition is quite rare, De Morgan observed: 'Many minds, and almost all uneducated ones, can hardly retain an intermediate state' regarding their beliefs. He thought that any query would do to prove this point: 'Put it to the first comer, what he thinks on the question whether there be volcanoes

[35] Jevons, *Pure Logic*, p.171.
[36] Ibid., p. 202.
[37] R. Harley, *Obituary Notices of the Royal Society*, no. 226 (Sept. 1883), p. xi.
[38] A. De Morgan, *Formal Logic* (London, 1847), p. 241.
[39] A. De Morgan, *Address to the London Mathematical Society* (London, 1865), p. 8.

on the unseen side of the moon larger than those on our side. The odds are, that though he has never thought of the question, he has a pretty stiff opinion in three seconds.'[40] De Morgan's *A Budget of Paradoxes* (1872) compiled cases of such mental shortcomings into an encyclopedic criticism of common modes of human thought.[41]

In his 1876 primer on logic Stanley Jevons spoke about the critical importance of removing the ambiguity endemic to so many of the words we use, often casually and without consideration, in both our everyday life and in the realms of speculative thought. 'Nothing indeed can be of more importance to the attainment of correct habits of thinking and reasoning than a thorough acquaintance with the great imperfections of language,' Jevons intoned at the beginning of the primer. 'Comparatively few terms have one single clear meaning and one meaning only, and whenever two or more meanings are unconsciously confused together, we inevitably commit a logical fallacy.'[42] What was needed, he believed, was greater attention to 'univocal' terms—words with just a single, well-defined meaning—rather than the far more common 'equivocal' terms—words that have more than one meaning and are thus inherently ambiguous. 'Equivocal terms are astonishingly common,' Jevons noted, 'They include most of the nouns and adjectives which are in habitual use in the ordinary intercourse of life.'[43] With this sort of ambiguity, how could it be possible to engage in Mill's 'loose kind of inference from particulars to particulars' with complete confidence?[44] It seemed to Jevons that human beings also needed a 'universal reasoning', such as the rigorous logic of the mathematicians Boole and De Morgan, which had been constructed without reference to particulars and in which all of the terms were definitively univocal.[45]

Jevons' concern about the frailty of human reasoning and its inferiority compared to symbolic logic went so far that he proposed replacing the actions of the human mind with a logical machine. In a paper celebrated in the annals of the history of computing, Jevons told a Royal Society of London audience in January 1870 about a new machine for the computation of logical conclusions. After a series of more prosaic terms he arrived at the evocative name 'logical piano' for his new machine, which he actually had built and which now sits in the Museum of the History of Science in Oxford. Jevons could

[40] De Morgan, *Formal Logic*, pp. 182–3.
[41] A. De Morgan, *A Budget of Paradoxes* (London, 1872).
[42] W. S. Jevons, *Elementary Lessons in Logic* (new edn., London and New York, 1895), pp. 27–8.
[43] Ibid., pp. 29–30.
[44] Jevons, *Pure Logic*, p. 203.
[45] Ibid.

scarcely contain his glee at the prospect that with the logical piano the faulty connections made by the human mind would be replaced by unmatched mechanical accuracy and sure conclusions. 'We have but to press a succession of keys in the order corresponding to the terms, conjunctions, and other parts of the propositions, in order to effect a complete analysis of the argument,' he happily told his audience, for 'the parts of the machine embody the conditions of correct thinking.'[46] Jevons's machine had an early 'enter' key—more artfully titled the '*finis* key'—that the operator pressed when he had completed entering the arguments and wished to explore the conclusions these arguments justified.

Like Jevons and De Morgan, John Venn similarly drew a contrast between conventional language, including the ordinary language of traditional logic, and a more scientific formulation that reduced the possibility for error. 'In ordinary life it is notorious that very many of the propositions to which the logician insists upon prefixing his bare "some", had really presented themselves with the more quantitative prefixes of "many" or "most" ', Venn cautioned. To sharpen and improve thought, language would have to be clarified with an eye on 'the universal and the definite'.[47] 'The intimate connexion between Language and Thought', he declared in his seminal book *The Principles of Empirical or Inductive Logic* (1889), 'is an abundantly sufficient ground for looking to some reform in the former as likely to afford powerful help towards advance in the latter.'[48] Venn's diagrammatical system of overlapping ellipses, a graphical version of Boole's mathematical system for encapsulating language and thus thought in a more precise, scientific way, was his attempt at such a reform. Methods for replacing language with clearer symbolic equivalents, like his own scheme or those of Boole or De Morgan, were not 'a mere system of shorthand', Venn emphasised; they were a means 'of improving at the fountain head both the ideas themselves and the methods of combining and analysing them'.[49] Moreover, like the contemporaneous introduction of 'universal' languages like Esperanto, he saw symbolic logic as part of a critical modern quest for improved communications across boundaries—a mission of great social importance. The diversity of the spoken word may not be as problematic as the inequality of wealth, he observed in *The Principles of Empirical or Inductive Logic*, but it is surely an 'evil' that must be addressed.[50]

[46] Jevons, *Pure Logic*, p. 170.
[47] J. Venn, *Symbolic Logic* (2nd edn., New York, 1894), p. 131.
[48] J. Venn, *The Principles of Empirical or Inductive Logic* (2nd edn., New York, 1907), p. 515.
[49] Ibid., p. 519.
[50] Ibid., p. 532.

Despite this emotional appeal that matched the early to mid-Victorian ecumenism and activism of Jevons, De Morgan, and Boole, Venn nevertheless felt more upbeat about the prospects for clarity and economy in language. Ambiguity of terms and its associated problems of weak reasoning and logical fallacies were giving way to a better age, he increasingly felt toward the turn of the century (notably well beyond the tragically shortened lifetimes of Boole and Jevons). It was an 'irresistible course of events', Venn believed, that human speech and thus thought would become clearer. 'As concerns subordinate departments of life, one class of communications after another is tending to the adoption of abbreviated symbols or conventional and artificially framed words for conveying widely recognized conceptions ... as was long ago noticed by Leibnitz in the parallel case of mathematical notation,' Venn concluded in *The Principles of Empirical or Inductive Logic*.[51] Given this optimism, Venn unsurprisingly felt much more charitable toward John Stuart Mill's notions of induction and even the traditional logic of the syllogism. If language was steadily improving, such linguistic methods might not have to be thrown out in favour of a purely mathematical system after all.

Beyond such concerns about human language and reasoning, the mathematical logicians, including Venn, had a darker worry about the implications of Mill's *Logic*, and indeed a broader worry about the knowledge of their age, mathematics included: that it might become so materialistic as to remove God from the universe. In this sense it is often difficult to separate discussions of mathematical knowledge in this period from more familiar discussions relating to the Victorian crisis of faith. It is noteworthy how often religious questions crept into Victorian mathematical works. Boole's seminal *Laws of Thought* (1854) had a critical chapter on the theology of Spinoza and Samuel Clarke; Venn's *Logic of Chance* (1866) delved extensively into questions of how we believe and concluded with a chapter on the credibility of miracle testimony; Jevons similarly ended his most important work, *The Principles of Science* (1874), with a heartfelt psalm on the place of the divine in an increasingly scientific world. With regard to mathematical logic and the foundations of mathematics in general, these Victorians had to come to an understanding of whether their symbolical containers and methods and the science they underwrote were exhaustive, or instead limited in applicability and thus accommodating of other, perhaps more transcendental, knowledge.

George Boole's lecture on the 'The Claims of Science', which inaugurated the difficult school year of 1851 at Queen's College, Cork, was typical of the relationship between faith and concerns about the nature of modern

[51] Ibid., p. 534.

scientific knowledge. Boole explored the worrisome prospect that science—broadly construed as formal human knowledge based on experience—might lead to scepticism or even atheism. 'If, before the time of Bacon, the external sources of human knowledge were too little regarded,' Boole declared, 'we may, in the strong reaction of a subsequent age against this form of error, discern perhaps too much of the contrary tendency.'[52] For Boole, a disciple of Kant and Plato with an affinity for philosophical idealism, it seemed improper for Mill's mode of induction to take excessive precedence over deduction from higher principles found in the mind. What his age needed, the mathematician conceived, was a healthy combination of induction and deduction—'the material and the mental'—in order for science to progress and serve humanity properly.[53] The laws of thought that Boole believed he had found using mathematical techniques were of the same universal nature and necessity as the path of a heavenly body, and he believed that both showed the guiding hand and design of God. 'Does the dominion of science terminate with the world of matter, or is there held out to us the promise of something like exact acquaintance, however less in extent, with the interior and nobler province of the mind?' Boole rhetorically asked his diverse audience.[54] He concluded his lecture by drawing a parallel between natural theology and the theology that he felt naturally evolved out of his mathematical laws of thought. 'With instances of mechanical adaptation in the works of the Divine Architect, we are all familiar. But to the reflective mind, there are few adaptations more manifest, there is none more complete, than that which exists between the intellectual faculties of man, and their scenes and occasions of exercise. Shall we not then confess that here also design is manifest?' Boole exclaimed.[55] Such a Kantian faith in the correspondence between the human mind and a divinely planned universe allowed Boole to ward off the spectre of sceptical materialism.

Boole's followers adopted both his symbolic logic and his interest in preserving a role for religious knowledge and belief, though they diverged somewhat from his divine conception of mathematical knowledge. Stanley Jevons made smaller claims about mathematics that he felt would reserve space for religion apart from science. As Jevons bluntly asserted in *The Principles of Science*, 'Even mathematicians make statements which are not true with absolute generality.'[56] Rather, Jevons believed, mathematical and

[52] Boole, *Studies in Logic*, p. 191.
[53] Ibid.
[54] Ibid., p. 194.
[55] Ibid., p. 199.
[56] Jevons, *The Principles of Science* (reprint of 1877 edn., London, 1924), p. 43.

scientific knowledge—even knowledge aided by the clear terms and rigour of symbolic logic—could only address a limited realm and generally did so with mere probability rather than certainty. Continuing a line of argument he began in his analysis of Mill's thought, Jevons impugned the workings of scientific induction, which supposedly used a series of observations to come to a 'certain' law:

> In the majority of cases it is impossible to collect together, or in any way to investigate, the properties of all portions of a substance or of all the individuals of a race. The number of objects would often be practically infinite, and the greater part of them might be beyond our reach, in the interior of the earth, or in the most distant parts of the Universe. In all such cases induction is *imperfect*, and is affected by more or less uncertainty... The powers of the human mind are so limited that multiplicity of detail is alone sufficient to prevent its progress in many directions.[57]

Jevons hoped that pure mathematics would show the way toward a more perfect form of induction, but the road would be difficult and the destination a realm where probability supplanted certainty. 'The whole question now becomes one of probability and improbability', Jevons remarked about this new realm of knowledge, 'We do not really leave the region of logic; we only leave that where certainty, affirmative or negative, is the result, and the agreement or disagreement of qualities the means of inference.'[58] As faith in scientific certainty waned, so would insidious materialism, Jevons believed, since it fed off arrogant and expansive theories of scientific knowledge.

Furthermore, the Unitarian Jevons maintained God's ultimate role in human knowledge, and even held out the possibility of His activity in the material world. 'We hang ever upon the will of the Creator: and it is only so far as He has created two things alike, or maintains the framework of the world unchanged from moment to moment, that our most careful inferences can be fulfilled,' Jevons declared in *The Principles of Science*.[59] We rely on God for the certainty that is by nature missing from the human mind's process of induction. At the conclusion of *The Principles of Science*, Jevons reiterated that scientific knowledge is limited, and thus unable to destroy transcendental beliefs. 'The conclusions of scientific inference appear to be always of a hypothetical and provisional nature,' he repeated emphatically, 'the best calculated results which it can give are never absolute probabilities; they are purely relative to the extent of our information.'[60] One can sense the genesis

[57] Ibid., pp. 146–8. Emphasis in the original.
[58] Ibid., p. 151.
[59] Ibid., p. 149.
[60] Ibid., p. 765.

of a familiar modern compromise here: science is impressive and powerful but ultimately limited in its purview and certainly not invalidating of religion, which functions in another sphere entirely. Thus the larger intent of Jevons's theoretical work became clear:

> My purpose ... is the purely negative one of showing that atheism and materialism are no necessary results of scientific method. From the preceding reviews of the value of our scientific knowledge, I draw one distinct conclusion, that we cannot disprove the possibility of Divine interference in the course of nature ... From science, modestly pursued, with a due consciousness of the extreme finitude of our intellectual powers, there can arise only nobler and wider notions of the purpose of Creation.[61]

Mathematics and symbolic logic had clearly helped Jevons come to this conclusion that science and materialism were not inextricably linked. As he noted, pure mathematics allows the mind to conceive and explore the infinite as well as entities that do not conform to the normal laws of space and time.[62] Such investigations inform the mind that the material realm might not be all that there is.

John Venn also used pure mathematics and probability theory in his attempt to understand certainty and belief, both in everyday life as well as in religion. He found it hard to conceive of pinpointing belief, however, on some kind of a mathematical spectrum. In day-to-day existence we are constantly swayed by hope and fear regarding the supposition to be believed, as well as a vast array of other emotions, Venn thought. Moreover, like Stanley Jevons he highlighted our imperfect knowledge of the world and our exceedingly limited experience.[63] 'The substructure of our convictions is not so much to be compared to the solid foundations of an ordinary building,' Venn strikingly wrote in *The Logic of Chance*, 'as to the piles of the houses of Rotterdam which rest somehow in a deep bed of soft mud.' With shades of David Hume's characterisation of personal identity in his *Treatise of Human Nature*, Venn continued, 'We are like a person listening to the confused hubbub of a crowd, where there is always something arbitrary in the particular sound we choose to listen to.'[64] The world is thus a bewildering realm of probability and fallibility.

[61] Jevons, *Principles of Science*, pp. 766–8.

[62] Ibid., pp. 767–8. Jevons noted on pp. 768, 'The study of logical and mathematical forms has convinced me that even space itself is no requisite condition of conceivable existence. Everything, we are told by materialists, must be here or there, nearer or further, before or after. I deny this, and point to logical relations as my proof.'

[63] Venn, *The Logic of Chance* (3rd edn., London, 1888), pp. 125–7.

[64] Ibid., p. 127.

Venn, who like many Victorian intellectuals was the product of an evangelical Church of England upbringing followed by periods of doubt (in part brought on by his reading of Mill's *Logic*), and who resigned from the clergy under the Clerical Disabilities Act in 1883 while maintaining an idiosyncratic faith, spent much of his life trying to comprehend how a heavenly realm could make itself known in the midst of such perplexity. His remarkable chapter 'On the Credibility of Extraordinary Stories' in *The Logic of Chance* shows Venn's questioning mind at its most nimble, as he retraced Hume's footsteps in analysing revealed religion using probability theory. On the one hand, Venn noted, people may be more careful when describing miracles since they are so unusual, and congruent descriptions of such experiences among many different people mathematically should have a higher probability of being true. On the other hand, he argued, how could one really measure the veracity of testimony and the character of witnesses? The pure mathematics of probability theory, with its parochial examples of coin tosses and lottery numbers, surely was hopelessly inadequate to the task of confirming or even assessing evidence of the divine.[65]

Such commonsensical conclusions slowly distanced John Venn philosophically from the mathematician he most wished to emulate, George Boole. By the late Victorian era, as Venn was writing his own *Symbolic Logic* (1881), it seemed naïve to believe that mathematical logic would have any serious affect on British society or perennial (and frequently predictable) religious disputes. Too often there is not enough common ground between antagonists on which to base moderating logical analysis, Venn realised. As early as his 1865 discussion of miracles in *The Logic of Chance*, Venn appreciated the radically different assumptions held by those on each side of the debate. Those who battled over the veracity of 'extraordinary stories' were separated by a 'chasm', where regardless of the format of the arguments—couched in mathematical terms or not—there was little chance for resolution. 'What is to be complained of in so many popular discussions on the subject is the entire absence of any recognition of the different ground on which the attackers and defenders of miracles are so often really standing,' he concluded.[66] Materialists and others who scorn revealed religion simply have an irreconcilable viewpoint from those who have faith in miracles, Venn thought: 'How therefore can miraculous stories be ... taken account of, when the disputants, on one side at least, are not prepared to admit their actual occurrence anywhere or at any time? How can any arrangement of bags and balls, or other

[65] Ibid., pp. 406–34, *passim*, esp. pp. 427–8.
[66] Ibid., p. 434.

mechanical or numerical illustrations of unlikely events, be admitted as fairly illustrative of miraculous occurrences?'[67] To Boole's ideal 'laws of thought,' universally found in every human mind, Venn therefore added a realist's understanding of human nature. We are stubborn, dogmatic, and indelibly coloured by our assumptions and limited experiences; no amount of mathematical reasoning appears capable of closing the gap between those with starkly different beliefs. For Venn, mathematical logic was thus more of a practical tool, albeit a powerful one, than a tool of social and transcendental significance.[68]

A year before the founding of the British Academy, Bertrand Russell extended and further refined Venn's understanding of symbolic logic with a sense of mathematics stridently devoid of extra-mathematical meanings. 'Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true,' Russell informed the lay readership of *The International Monthly* in 1901.[69] He meant this as a compliment, of course, not a criticism: mathematical processes derive their power solely from the clarity of their terms and the rigour of their logic, with no external referents; mathematicians can develop methods that are internally consistent and divorced from any specific content, goal, application, or agenda. In its most pure form mathematics simply involved the containers into which we could place a variety of things, and the ways we can manipulate those containers, while setting aside an interest in the things themselves. From this perspective a purely technical history of mathematics in the Victorian age is perhaps all that is needed.

[67] Venn, *Logic of Chance*, p. 424.
[68] Venn, *Symbolic Logic*, pp. xviii–xxvii.
[69] B. Russell, 'Recent work on the principles of mathematics', *The International Monthly*, 4 (1901), pp. 83–101, at p. 84.